Pergamon

www.elsevier.com/locate/jappmathmech

PII: S0021-8928(99)00036-2

J. Appl. Maths Mechs, Vol. 63, No. 2, pp. 277–281, 1999

0021-8928/99/$—see front matter

A SHEAR SURFACE WAVE AT THE INTERFACE OF AN ELASTIC BODY AND A MICROPOLAR LIQUID†

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(*Received* 28 *April* 1998)

The propagation of a shear surface wave (SSW) along the interface of an elastic half-space liquid and micropolar half-space is considered. The phase velocity of the wave and its attenuation constant are determined. It is shown that a SSW at the interface of a solid and a micropolar liquid propagates almost without dispersion, unlike a wave at the interface of a solid and a viscous Newtonian liquid in which the dispersion is noticeable at high frequencies. The attenuation of the SSW is considerably weaker in the first case than the second. The attenuation constant decreases as the boundary viscosity increases.

It is well known that there can be a slightly non-uniform attenuating shear wave at the interface of an elastic medium and a viscous liquid; its properties have been studied in some detail [1–3]. The features of the propagation of a SSW in the upper megahertz and gigahertz bands can be influenced by the microstructure of both the solid and the liquid. We will confine ourselves here to investigating the influence of the micropolarity of the liquid on the SSW.

We introduce an orthogonal Cartesian system of coordinates in which the xy plane coincides with the interface of the two media. The z axis is directed into the micropolar liquid.

The wave process in an isotropic linearly-elastic half-space can be described by the equation [4]

$$\mathbf{u}_{tt} - c_l^2 \operatorname{grad} \operatorname{div} \mathbf{u} + c_\tau^2 \operatorname{rot} \operatorname{rot} \mathbf{u} = 0 \tag{1}$$

where $\mathbf{u}$ is the displacement vector, c_l is the velocity of dilatation waves and c_τ is the velocity of shear waves. In a viscous micropolar incompressible liquid the wave motion can be described in the linear approximation by the equations [5–7]

$$\begin{aligned} &\mathbf{V}_t = -\frac{\nabla p}{\rho_f} + (\nu + \nu_r)\Delta \mathbf{V} + 2\nu_r \operatorname{rot} \boldsymbol{\omega} \\ &I\boldsymbol{\omega}_t = 2\nu_r(\operatorname{rot} \mathbf{V} - 2\boldsymbol{\omega}) + \gamma\Delta\boldsymbol{\omega} + (\vartheta + \delta)\operatorname{grad} \operatorname{div} \boldsymbol{\omega} \\ &\operatorname{div} \mathbf{V} = 0 \end{aligned} \tag{2}$$

where $\mathbf{V}$ is the velocity vector, ρ_f is the density of the liquid, p is the pressure, I is a scalar constant with the dimension of moment of inertia of unit of mass, ω is the microrotation vector, ν is the coefficient of kinematic Newtonian shear viscosity and ν_r, ϑ, δ, γ are the coefficients of moment viscosity.

Using the representation for the microrotation vector ω in terms of Lamb potentials

$$\boldsymbol{\omega} = \nabla\Theta + \operatorname{rot} \boldsymbol{\psi}, \quad \operatorname{div} \boldsymbol{\psi} = 0 \tag{3}$$

assuming that the wave propagates along the x axis and the particles move along the y axis perpendicular to the propagation vector, we can reduce the initial vector equations (1) and (2) to four scalar equations

$$u_{tt} = c_\tau^2 \Delta_\perp u \tag{4}$$

$$V_t = (\nu + \nu_r)\Delta_\perp V - 2\nu_r \Delta_\perp \psi \tag{5}$$

$$I\psi_t = 2\nu_r(V - 2\psi) + \gamma\Delta_\perp \psi \tag{6}$$

$$I\Theta_t = -4\nu_r\Theta + (\vartheta + \gamma + \delta)\Delta_\perp\Theta, \quad \Delta_\perp = \partial_x^2 + \partial_z^2 \tag{7}$$

†*Prikl. Mat. Mekh.* Vol. 63, No. 2, pp. 289–294, 1999.

Since all the quantities depend on only two space variables, we can use the notation Δ instead of $\Delta_\perp$. It is useful to replace Eqs (5) and (6) by equations with only one dependent variable

$$LV=0,\ L\psi=0 \tag{8}$$
$$L=I\partial_t^2-4\nu\nu_r\Delta+\gamma(\nu+\nu_r)\Delta^2-\left(I(\nu+\nu_r)+\gamma\right)\partial_t\Delta+4\nu_r\partial_t$$

At the interface ($z=0$), we have

$$u_t=V \tag{9}$$

$$\rho_s c_\tau^2 u_z=\rho_f(\nu-\nu_r)V_z-2\nu_r\rho_f(\Theta_x-\psi_z) \tag{10}$$

$$2(\Theta_x-\psi_z)+\alpha V_z=0,\quad 2(\Theta_z-\psi_x)-\alpha V_x=0 \tag{11}$$

$$\delta\Delta\Theta+(\gamma+\vartheta)(\Theta_{zz}+\psi_{xz})=0,\quad (\vartheta+\gamma)\Theta_{xz}+\vartheta\psi_{xx}-\gamma\psi_{zz}=0 \tag{12}$$

Here α can take any value in the half-segment (01] and ρ_s is the density of the elastic medium. Relation (9) is the condition for continuity of the displacement velocities, (10) is the condition for continuity of the shear stresses, (11) is the condition for proportionality of the micro-rotation vector to the mean angular velocity of particles of the liquid medium and (12) is the condition for the moment stress to vanish.

Taking the first equation of (1) into account, it is more convenient to write boundary condition (10) in the form

$$\rho_s c_\tau^2 u_z=\rho_f(\nu-\nu_r+\nu_r\alpha)V_z \tag{13}$$

It also makes sense to use (6) to convert the second boundary condition of (12) to the form

$$(\vartheta+\gamma)(\Theta_{xz}+\psi_{xx})-I\psi_t+2\nu_r(V-2\psi)=0 \tag{14}$$

We shall seek a solution of system (4)–(7) in the form of a travelling non-uniform wave (c.c. denotes the complex conjugate of the expression that precedes it)

$$\begin{aligned}
&u=\hat U\exp(-i\omega t+ikx+\lambda z)+\text{c.c.},\quad \lambda^2=k^2-\frac{\omega^2}{c_\tau^2}\\
&V=\sum_{j=1}^{2}\hat V_j\exp(-i\omega t+ikx-\lambda_j z)+\text{c.c.}\\
&\psi=\sum_{j=1}^{2}\hat\psi_j\exp(-i\omega t+ikx-\lambda_j z)+\text{c.c.}\\
&\Theta=\hat\Theta\exp(-i\omega t+ikx+\lambda_3 z)+\text{c.c.},\quad \lambda_3^2=k^2+\frac{4\nu_r-i\omega I}{\gamma+\delta+\vartheta}
\end{aligned} \tag{15}$$

$$\lambda_j^4+\left(-2k^2+i\omega\frac{I(\nu+\nu_r)+\gamma}{\gamma(\nu+\nu_r)}-\frac{4\nu\nu_r}{\gamma(\nu+\nu_r)}\right)\lambda_j^2+k^4+\frac{1}{\gamma(\nu+\nu_r)}\times$$
$$\times\left\{-I\omega^2-4i\nu_r\omega-i\left[I(\nu+\nu_r)+\gamma\right]\omega k^2+4\nu\nu_r k^2\right\}=0,\quad j=1,2 \tag{16}$$

where k is the complex wave number and $\operatorname{Re}\lambda>0$, $\operatorname{Re}\lambda_3>0$, $\operatorname{Re}\lambda_j>0$ ($j=1,2$), $\hat V_j$, $\hat\psi_j$, $\hat\Theta$ are constants. Here and below the symbol ω is used only to denote the angular frequency.

Substituting the harmonic travelling wave (17) into the boundary conditions (9), (11), (13), (14) and the first condition of (12), we obtain

$$\begin{aligned}
&-i\omega\hat U=\hat V_1+\hat V_2\\
&\rho_s c_\tau^2\lambda\hat U=-\rho_f(\nu-\nu_r+\alpha\nu_r)(\lambda_1\hat V_1+\lambda_2\hat V_2)\\
&2(ik\hat\Theta+\lambda_1\hat\psi_1+\lambda_2\hat\psi_2)-\alpha(\lambda_1\hat V_1+\lambda_2\hat V_2)=0
\end{aligned} \tag{17}$$

$$2\left[-\lambda_3\hat{\Theta} + ik(\hat{\psi}_1 + \hat{\psi}_2)\right] - i\alpha k(\hat{V}_1 + \hat{V}_2) = 0$$

$$\delta(\lambda_3^2 - k^2)\hat{\Theta} + (\gamma + \vartheta)\left[\lambda_3^2\hat{\Theta} - ik(\lambda_1\hat{\psi}_1 + \lambda_2\hat{\psi}_2)\right] = 0$$

$$ik\lambda_3(\gamma + \vartheta)\hat{\Theta} + \left[k^2(\gamma + \vartheta) + 4\nu_r - i\omega I\right](\hat{\psi}_1 + \hat{\psi}_2) - 2\nu_r(\hat{V}_1 + \hat{V}_2) = 0$$

Note that the most common condition used for the microrotation vector is that it should equal zero at a solid wall, which corresponds to α in formulae (11) taking the value zero. However, when $\alpha = 0$, analysis of boundary conditions (17) shows that there cannot be a SSW for any values of the parameters of the contacting elastic half-space and micropolar liquid, which contradicts what we know from acoustics.

From the condition for the five linear homogeneous equations (17) to be compatible, cumbersome but standard calculations yield a dispersion equation for the SSW. The following method is simpler. Substituting the expression for $\hat{\psi}_1 + \hat{\psi}_2$ from the fourth equation of (17) into the last equation, we obtain a relation between $\hat{V}_1$, $\hat{V}_2$ and $\hat{\Theta}$. From this and the equation obtained from the fifth condition (17) with the help of the third condition, we eliminate $\hat{\Theta}$. The result is an equation which contains only $\hat{V}_1$ and $\hat{V}_2$

$$2(\lambda_3^2 - k^2)(\gamma + \vartheta + \delta)\left[k^2(\gamma + \vartheta) + 4\nu_r(1 - \alpha^{-1}) - i\omega I\right](\hat{V}_1 + \hat{V}_2) +$$

$$+\lambda_3(4\nu_r - i\omega I)(\gamma + \vartheta)(\lambda_1\hat{V}_1 + \lambda_2\hat{V}_2) = 0 \tag{18}$$

The first two conditions of (17) enable us to write one more equation in $\hat{V}_1$, $\hat{V}_2$

$$\rho_s c_\tau^2 \lambda(\hat{V}_1 + \hat{V}_2) = i\omega\rho_f(\nu - \nu_r + \alpha\nu_r)(\lambda_1\hat{V}_1 + \lambda_2\hat{V}_2) \tag{19}$$

Equations (18) and (19) are solvable for $\hat{V}_1 + \hat{V}_2$, $\lambda_1\hat{V}_1 + \lambda_2\hat{V}_2$ under a condition which, using the expression for λ_3 from (15), we convert to the form

$$\rho_s c_\tau^2 \lambda = -i\omega\rho_f(\nu - \nu_r + \alpha\nu_r)\lambda_3^{-1}\left[k^2 + \frac{4\nu_r(1 - \alpha^{-1})}{\gamma + \vartheta} - \frac{i\omega I}{\gamma + \vartheta}\right] \tag{20}$$

Note that the dispersion equation for a surface shear wave is much simpler in the case of a viscous Newtonian liquid

$$\rho_s c_\tau^2 \lambda - i\omega\rho_f \nu\left(k^2 - \frac{i\omega}{\nu}\right) = 0$$

The first term ($k^2 = O(\omega^2/c_\tau^2)$) and the third term in square brackets on the right-hand side of Eq. (20) are much smaller than the second term for waves in the megahertz band, and hence can be neglected. This is also true of the terms in the expression for λ_3^2. Neglecting small terms and squaring both sides of Eq. (20), it can be solved for k^2. After simple calculations, we obtain expressions for the phase velocity and attenuation constant of the SSR.

We introduce the dimensionless terms

$$r = \frac{\rho_f}{\rho_s}, \quad g = \frac{\nu_r(\nu - \nu_r + \alpha\nu_r)^2}{c_\tau^2(\gamma + \vartheta)}, \quad h = 1 + \frac{\delta}{\gamma + \vartheta}$$

$$\Omega = \frac{\nu\omega}{c_\tau^2}, \quad f = \frac{c_\tau^2 I}{\nu^2}, \quad e = \frac{\nu_r}{\nu}$$

Then the expressions for the phase velocity c and the attenuation constant of the SSW will take the form

$$c \approx c_\tau\left(1 + 2r^2 g(\alpha)h(1 - \alpha^{-1})^2\right)$$

$$\beta \approx \tfrac{1}{2}\nu^{-1}c_\tau\Omega^2 r^2 h(\alpha^{-2} - 1)e^{-1}gf \tag{21}$$

It follows from the first formula of (21) that SSW propagates practically without dispersion at the interface of a solid and a micropolar liquid in the megahertz band, unlike the wave at the interface of a solid and a viscous Newtonian liquid, where the dispersion is more noticeable at those frequencies

$$c_n \approx c_\tau\left(1 + \tfrac{1}{8}\Omega^2(4 - r^2)r^2\right)$$

where c_n is the phase velocity of the SSW at the interface of the elastic half-space and a viscous Newtonian liquid. According to published data, the dimensionless term g is of the order of $\approx 10^{-11}(1 + \alpha)^2$, and so the difference between the phase velocity of a SSW and that of a shear wave c_τ is exceptionally small. The difference between the two is much greater for a viscous Newtonian liquid. The micropolar properties reduce the depth to which the exponentially attenuating wave field of the SSW penetrates the liquid and at the same time lessen the difference between the non-uniform wave field of the SSW in an elastic body and a plane shear wave. Comparing the expression for the attenuation constant β in (21) and that for β_n of SSW along the interface of an ordinary viscous fluid and an elastic body

$$\beta_n \approx \tfrac{1}{2}\nu^{-1}c_\tau r^2\Omega^2$$

we see that the SSW attenuates considerably more weakly in the case of a moment liquid.

The coefficient α, associated with the boundary viscosity of a micropolar fluid,† has a large effect on the velocity of the SSW (although this is small compared with c_τ), especially in cases when the rotational viscosity ν_r is comparable in value with the ordinary coefficient of kinematic viscosity ν. The relation between $c/c_\tau - 1$ and the parameter α is shown in Fig. 1 when g takes the values $g_1 = 0.5 \times 10^{-11}(1 + \alpha)^2$, $g_2 = 0.4 \times 10^{-10}(0.9 + 0.1\alpha)^2$, $g_3 = 0.8 \times 10^{-11}(1 + 0.2\alpha)^2$, corresponding to values of e equal to 0.5; 0.1; 0.02, while the other parameters are unchanged: $r = 0.13$, $f = 2.3 \times 10^3$, $h = 1$. The frequency dependence of the attenuation constant $\nu\beta/c_\tau$ for the same parameter values and $g = g_1$ is also shown. It is interesting to note that the attenuation constant decreases as the boundary viscosity increases, that is, as α increases.

For waves in the gigahertz band, the first term in square brackets on the right-hand side of Eq. (20) becomes much greater than the second. In that case the expressions for the phase velocity and attenuation constant take the form

$$c \approx c_\tau\left[1 + \tfrac{1}{2}\Omega^2 r^2(1 - (1-\alpha)e)^2\right]$$
$$\beta \approx \tfrac{1}{2}\nu^{-1}c_\tau\Omega^2 r^2 g(\alpha) f e^{-1}(2 - h^{-1}) \qquad (22)$$

Thus, for a SSW at the interface of an elastic body and a micropolar liquid the frequency curves of the phase velocity and attenuation constant are quadratic as in the case of a Newtonian liquid. It is worth mentioning that formulae (22) may also hold at lower frequencies if α takes a value close to unity. Whether this can actually happen is still unclear.

Indirect experimental results suggest that the microrotational moment of inertia l is no greater than 10^{-16} m^2. This might not be a completely reliable result. There could be micropolar liquids with a large constant l of the order of 10^{-12} m^2. For such "strongly microinertial" liquids, the expressions for the phase velocity and attenuation constant of the SSW would take the form

$$c \approx c_\tau\left[1 - \tfrac{1}{8}\Omega^2 f^2 e^{-2} r^2 g^2(\alpha) h^2\right]$$
$$\beta \approx \tfrac{1}{2}\nu^{-1}c_\tau\Omega^2 r^2 g(\alpha) f e^{-1} h$$

This research was supported financially by the Russian Foundation for Basic Research (96-02-19792), a grant from the President of the Russian Federation (98-15-96127) and the Federal Special-Purpose Programme "State Support for Integration of Higher Education and Fundamental Science" (No. 542).

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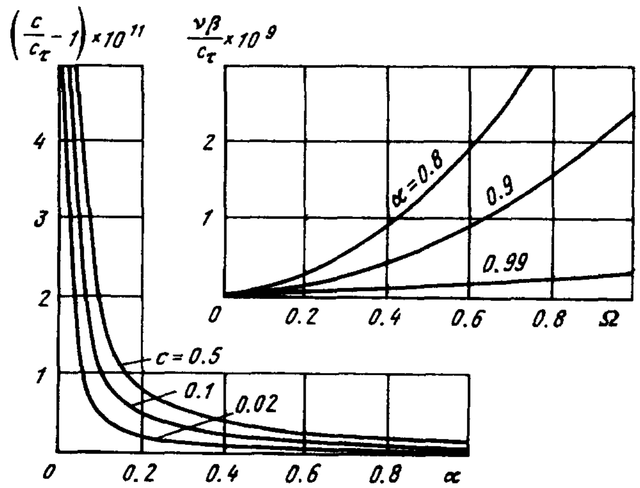

Fig. 1.

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Translated by R.L.